Journal of Engineering Physics and Thermophysics, Vol. 66, No. 3, 1994

OPTIMUM HEATING OF METAL IN CHAMBER FURNACES WITH MINIMUM OXIDATION

V. B. Kovalevskii and Kho Zhuitian'

UDC 621.1.0.18

An algorithm is developed that makes it possible to establish regimes for heating billets in a chamber furnace that provide the minimum amount of scale, and a numerical example is given.

The issues of economic efficiency, namely, of choosing the optimum technological regimes according to prescribed criteria, are among the most important issues in studying the operation of heating furnaces. In solving the problem of optimum control of metal heating according to the minimum amount of scale, effective algorithms are developed to optimize the operation of continuous furnaces [1–4]. In the present work we consider a similar problem as applied to chamber-type furnaces. The investigative procedure actually coincides with that proposed in [1-4].

Let the process of change of the temperature of the heating medium and the metal be described by the following equations [5, 6]:

$$\frac{dT_{\text{heat}}}{dt} = A_1 U - A_2 T_{\text{heat}} - A_3 (T_{\text{heat}} - T), \quad \frac{dT}{dt} = \mu (T_{\text{heat}} - T) \tag{1}$$

with the initial conditions

$$T_{\text{heat}}(0) = T_{\text{heat}_0}, \quad T(0) = T_0 \tag{2}$$

and the boundary conditions

$$T(t_k) = T_k . \tag{3}$$

From the condition of attainability of the metal temperature T_k and the physical limitations on fuel rate we assume that [5, 6]

$$0 < T_0 < T_k < \beta, \quad t_k \geq t_{\min}, \quad A_3 > A_2, \quad \frac{A_1}{A_2} U_h < T_k, \tag{4}$$

$$0 < U_h \leq U(t) \leq U_k, \quad 0 \leq t \leq t_k . \tag{5}$$

We introduce into consideration a quality criterion that determines the amount of scale at the end of heating [7]:

$$I = \int_0^{t_k} \frac{\alpha}{T(t)} \exp(-\beta / T(t))\, dt . \tag{6}$$

The problem of optimum control consists in choosing the regime of variation of the fuel rate in time $U(t)$ ($0 \leq t \leq t_k$) in the form of a piecewise continuous function satisfying condition (5) that minimizes functional (6) on solutions of (1)-(3).

Belarussian State Polytechnic Academy, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 66, No. 3, pp. 358-362, March, 1994. Original article submitted November 26, 1992.

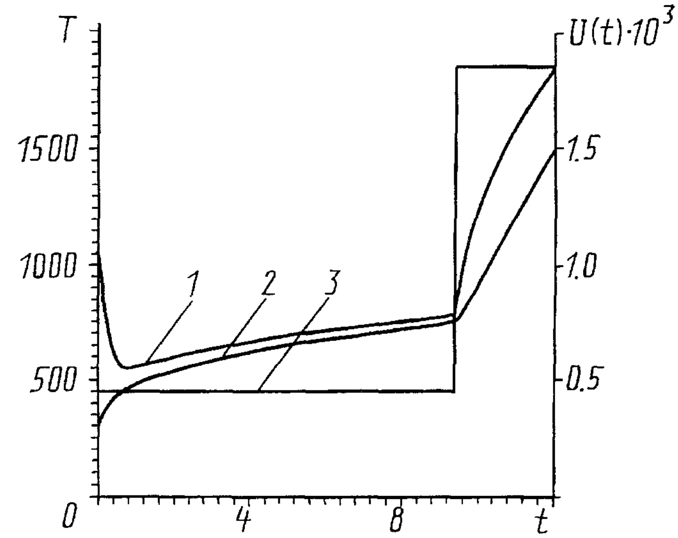

Fig. 1. Plots of optimum heating of billets: 1) medium temperature; 2) metal temperature; 3) fuel rate. T, K; t, h; U, m^3/h.

Using the body of mathematics of [1, 2, 8] we obtain the optimum trajectory of metal heating, the optimum fuel rate, and the moment of switching of the controlling effect (the fuel rate). The procedure for revealing the structure of the controlling function and the optimum trajectory and an algorithm for solving problem (1)-(6) are given in the Appendix.

Once the time of switching t_2 is found, the optimum fuel rate is written as

$$U(t) = \begin{cases} U_h, & 0 \le t \le t_2, \\ U_k, & t_2 < t \le T_k. \end{cases} \tag{7}$$

To solve this problem numerically, a program for a personal computer was written that made use of a method of dividing the segment $[0, t_k]$ in half.

Numerical data were chosen as follows: $\mu = 0.8$ 1/h, $A_1 = 1.3176$ K/m^3, $A_2 = 0.6516$ 1/h; $A_3 = 3.0492$ 1/h; $t_k = 12$ h; $T_0 = 300$ K; $T_k = 1500$ K; $T_{\text{heat}} = 1050$ K; $U_h = 450$ m^3/h; $U_k = 1850$ m^3/h; $\alpha = 2000$; $\beta = 3000$.

Figure 1 gives the time variations of the optimum temperature of the heating medium and the metal as well as the fuel rate. The switching time is $t_2 = 9.375$ h and the minimum amount of scale is 0.541 kg/m^2.

Thus, the regime for heating a metal to a prescribed temperature that provides the minimum amount of scale is characterized by two time intervals where the fuel rate is respectively minimum and maximum.

APPENDIX

We write system (1) in matrix form and find its solution. We have

$$T = AT + Q, \tag{8}$$

where

$$T = \begin{pmatrix} T_{\text{heat}}(t) \\ T(t) \end{pmatrix}, \quad A = \begin{pmatrix} -A_2 - A_3 & A_3 \\ \mu & -\mu \end{pmatrix}, \quad Q = \begin{pmatrix} A_1 U(t) \\ 0 \end{pmatrix}.$$

The determinant of the characteristic matrix is equal to

$$|\lambda E - A| = \begin{vmatrix} \lambda + A_2 + A_3 & -A_3 \\ -\mu & \lambda + \mu \end{vmatrix} = \lambda^2 + (A_2 + A_3 + \mu)\lambda + A_2\mu.$$

Since $A_3 > A_2$ (see (4)) the eigenvalues will be the real numbers and

$$\lambda_1 = \frac{-(\mu + A_2 + A_3) + \sqrt{(\mu + A_2 + A_3)^2 - 4A_2\mu}}{2},$$

$$\lambda_2 = \frac{-(\mu + A_2 + A_3) - \sqrt{(\mu + A_2 + A_3)^2 - 4A_2\mu}}{2}.$$

It is obvious that $0 > \lambda_1 > \lambda_2$.

It is easy to obtain the corresponding eigenvectors of system (8):

$$H_1 = \begin{pmatrix} \dfrac{\lambda_1 + \mu}{\mu} \\ 1 \end{pmatrix}, \quad H_2 = \begin{pmatrix} \dfrac{\lambda_2 + \mu}{\mu} \\ 1 \end{pmatrix}.$$

Thus, we obtain the solution to system (8) on the segment $[t_c, t]$ [6]:

$$T(t) = \exp(A(t - t_c))\, T_c + \int_{t_c}^{t} (\exp(A(t - \tau))\, Q)\, d\tau =$$

$$= \exp(A(t - t_c))\, T_c + \int_{t_c}^{t} (P \exp(B(t - \tau))\, P^{-1} Q)\, d\tau .$$

Here

$$T_c = \begin{pmatrix} T_{\text{heat}}(t_c) \\ T(t_c) \end{pmatrix}, \quad P = (H_1 \quad H_2) = \begin{pmatrix} \dfrac{\lambda_1 + \mu}{\mu} & \dfrac{\lambda_2 + \mu}{\mu} \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

t_c is an arbitrary time, $0 \le t_c \le t_k$.

Hence, we obtain

$$T_{\text{heat}}(t) = \frac{(\mu + \lambda_1)(\mu T_{\text{heat}}(t_c) - (\mu + \lambda_2)\, T(t_c)) \exp(\lambda_1 (t - t_c))}{(\lambda_1 - \lambda_2)\, \mu} +$$

$$+ \frac{(\mu + \lambda_2)(T(t_c)(\mu + \lambda_1) - \mu T_{\text{heat}}(t_c)) \exp(\lambda_2 (t - t_c))}{(\lambda_1 - \lambda_2)\, \mu} +$$

$$+ \int_{t_c}^{t} \frac{A_1 U (\exp(\lambda_1 (t - \tau))(\mu + \lambda_1) - \exp(\lambda_2 (t - \tau))(\mu + \lambda_2))}{\lambda_1 - \lambda_2}\, d\tau , \tag{9}$$

$$T(t) = \frac{(\mu T_{\text{heat}}(t_c) - (\mu + \lambda_2)\, T(t_c)) \exp(\lambda_1 (t - t_c))}{\lambda_1 - \lambda_2} +$$

$$+ \frac{(T(t_c)(\mu + \lambda_1) - \mu T_{\text{heat}}(t_c)) \exp(\lambda_2 (t - t_c))}{\lambda_1 - \lambda_2} +$$

$$+ \int_{t_c}^{t} \frac{\mu A_1 U (\exp(\lambda_1 (t - \tau)) - \exp(\lambda_2 (t - \tau)))}{\lambda_1 - \lambda_2}\, d\tau . \tag{10}$$

From (10) it follows that the lower the fuel rate $U(t)$, the lower the metal temperature $T(t)$.

By virtue of Corollary 1 obtained in [1] we have that the control $U_w(t)$ that realizes the infinite optimum trajectory (IOT) is a step function.

We will determine the IOT. For this purpose we estimate the derivative of the integrand of functional (6) $F(T) = \alpha/T_2 \exp(-\beta/T)$. We have

$$\dot{F}(T) = \frac{\alpha}{T^2} \exp(-\beta/T) \left(\frac{\beta}{T} - 1 \right) > 0, \quad T \in [T_0, A_2],$$

and therefore $F(T_1) \geq F(T_2)$ for $T_1 \geq T_2 \geq T_0$, i.e., the lower the metal temperature, the smaller the value of the integrand.

By $W_{\text{heat}}(t)$ and $W(t)$ we denote the trajectory that is found as a result of integrating Eqs.(1) for $U(t) = U_h$ with initial conditions (3).

For any permissible process of problem (1)-(5) $T_{\text{heat}}(t)$, $T(t)$, $U(t)$ it is proved that

$$T(t) \geq W(t), \quad t \in [0, t_k],$$

and therefore $F(T(t)) \geq F(W(t))$, $t \in [0, t_k]$,

$$\int_{t_1}^{t_2} F(T(t))\, dt \geq \int_{t_1}^{t_2} F(W(t))\, dt, \quad 0 \leq t_1 \leq t_2 \leq t_k.$$

So, we have proved that $W_{\text{heat}}(t)$, $W(t)$ are the IOT, $U_w(t) = U_h$ [1]. Problem (1) degenerates and $t_1 = 0$ [1]. We find the solution to the problem with a free left end of the trajectory $\Phi_{\text{heat}}(t)$, $\Phi(t)$ and the time t_2.

For $U(t) = U_h$, $t_c = 0$, $T(t_c) = T_0$, $T_{\text{heat}}(t_c) = T_{\text{heat}_0}$ we easily obtain from (10) that $\lim_{t \to \infty} W(t) = A_1/A_2\, U_h < T_k$ (see (4)), and consequently, there is an instant $t_2 < t_k$ such that differential equations (1) with the boundary conditions $T(t_2) = W(t)$, $T_{\text{heat}}(t_2) = W_{\text{heat}}(t_2)$, and $T(t_k) = T_k$ on the segment $[t_2, t_k]$ have a solution $\Phi_{\text{heat}}(t)$, $\Phi(t)$ such that for $t_2^1 > t_2$ the analogous boundary problem has no for every permissible regime of variation of the fuel rate.

The trajectory $\Phi_{\text{heat}}(t)$, $\Phi(t)$ can be obtained as a solution to the following problem:

$$\frac{d\Phi_{\text{heat}}}{dt} = A_1 U_k - A_2 \Phi_{\text{heat}} - A_3 (\Phi_{\text{heat}} - \Phi), \quad \frac{d\Phi}{dt} = \mu (\Phi_{\text{heat}} - \Phi), \tag{11}$$

$$\Phi(t_2) = W(t), \quad \Phi_{\text{heat}}(t_2) = W_{\text{heat}}(t_2). \quad \Phi(t_k) = T_k, \quad t_2 \leq t \leq t_k. \tag{12}$$

We show that $\Phi_{\text{heat}}(t)$, $\Phi(t)$, $t_2 \leq t \leq t_k$ and t_2 are the sought trajectory and time. Let $X_{\text{heat}}(t)$, $X(t)$ be the optimum solution to the corresponding problem with a free left end of the trajectory on the segment $[t_2, t_k]$; then $X(t_2) > \Phi(t_2)$ since in the opposite case this contradicts the choice of the time t_2. By virtue of the continuity of the functions $X(t)$ and $\Phi(t)$ on the segment $[t_2, t_k]$ and from (11) and (12) it follows that we find $t^1 < t_k$: $X(t^1) = \Phi(t^1)$. It is obvious that $X(t) = \Phi(t)$, $t_k \geq t \geq t^1$ and $X(t) = \Phi(t)$, $t^1 > t \geq t_2$. Hence we have

$$\int_{t_2}^{t_k} \frac{\alpha}{\Phi(t)} \exp(-\beta/\Phi(t))\, dt < \int_{t_2}^{t_k} \frac{\alpha}{X(t)} \exp(-\beta/X(t))\, dt,$$

i.e., $X_{\text{heat}}(t)$, $X(t)$ are not the optimum solution to the problem with a free left end of the trajectory on the segment $[t_2, t_k]$. We have obtained a contradiction. Consequently, $\Phi_{\text{heat}}(t)$ and $\Phi(t)$ are the optimum solution to the problem with a free left end of the trajectory.

Thus, the algorithm for solving problem (1)-(6) consists in determining the time t_2 which is found as the root of the equations

$$W_{\text{heat}}(\tau) - \Phi_{\text{heat}}(\tau) = 0\,, \quad W(\tau) - \Phi(\tau) = 0\,. \tag{13}$$

To find the root, we use a numerical method. The main idea consists in finding the instant t_2 and then the temperatures $W_{\text{heat}}(t_2)$ and $W(t_2)$. The algorithm can be as follows:

Step 1. We prescribe the accuracy of calculations ε and assume that $a = 0$ and $b = t_k$.

Step 2. We calculate $\tau = (a + b)/2$.

Step 3. For $U = U_h$ and $t_c = 0$ we calculate $W_{\text{heat}}(t)$ and $W(t)$ from (9) and (10).

Step 4. For $U = U_k$ and $t_c = \tau$ we calculate $\Phi(t_k)$ from (10).

Step 5. If $\Phi(t_k) > T_k + \varepsilon$ then $a = \tau$ and we pass to step 2.

Step 6. If $\Phi(t_k) < T_k - \varepsilon$ then $b = \tau$ and we pass to step 2.

Step 7. If $|\Phi(t_k) - T_k| \le \varepsilon$ we assume that $t_2 = \tau$ and we finish the calculations.

NOTATION

t, time; $T_{\text{heat}}(t)$, $T(t)$, temperatures of the heating medium and the metal respectively; $U(t)$, fuel rate at the instant t; t_k, time of termination of heating; μ, A_1, A_2, A_3, positive constants characterizing the dynamics of the heating process; U_h, U_k, minimum and maximum fuel rates, respectively; α, β, prescribed constants characterizing the dynamics of metal oxidation; $t_{\min}$, minimum time of heating of the metal from the temperature T_0 to T_k; T_0, T_k, initial and final metal temperature, respectively.

REFERENCES

1. V. B. Kovalevskii, V. I. Panasyuk, and O. Yu. Sedyako, Inzh.-Fiz. Zh., **59**, No. 1, 168-169 (1990).
2. V. B. Kovalevskii, F. M. Babushkin, and O. Yu. Sedyako, Inzh.-Fiz. Zh., **61**, No. 2, 199-203 (1991).
3. V. B. Kovalevskii, O. Yu. Sedyako, and V. S. Timofeev, Stal', No. 11, 50-52 (1991).
4. F. M. Babushkin, V. B. Kovalevskii, and V. I. Timoshpolskii, Izv. Vyssh. Uchebn. Zaved., Energet., No. 1, 105-108 (1992).
5. G. V. Trubitsyn and G. V. Sotnikov, Izv. Vyssh. Uchebn. Zaved., Chern. Metall., No. 12, 103-107 (1983).
6. G. V. Trubitsyn, Izv. Vyssh. Uchebn. Zaved., Chern. Metall., No. 3, 136-138 (1982).
7. A. G. Butkovskii, S. A. Malyi, and Yu. N. Andreev, Control of Metal Heating [in Russian], Moscow (1981).
8. L. S. Pontryagin, Ordinary Differential Equations [in Russian], Moscow (1970).